

6.3.4 Examples:**Example (1):**

The various axial loads for a building column have been computed according to the applicable building code with the following results:

dead load = 850 kN, load from roof = 250kN (roof live load), live load from floor = 1100 kN, compression wind = 350 kN, tensile wind = 290 kN, compression earthquake = 260 kN, and tensile earthquake = 300 kN.

Determine the ultimate load on the column.

Solution:

$D = 850$ kN, $F = 0$ kN, $T = 0$ kN, $L = 1100$ kN, $L_r = 250$ kN, $H = 0$ kN, $S = 0$ kN, $R = 0$ kN, $W = +350$ kN (Compression) and -290 kN (Tensile), $E = +260$ kN (Compression) and -300 kN (Tensile). After including all zero values of the loads in the load combinations, and expanding the equations to their permutations, the equations are reduced to:

$$1.4D$$

$$1.2D + 1.6L + 0.5L_r$$

$$1.2D + 1.6L_r + L$$

$$1.2D + 1.6L_r + 0.5W_{\text{tension}}$$

$$1.2D + 1.6L_r + 0.5W_{\text{compression}}$$

$$1.2D + 1.0W_{\text{tension}} + 1.0L + 0.5L_r$$

$$1.2D + 1.0W_{\text{compression}} + 1.0L + 0.5L_r$$

$$1.2D + 1.0L$$

$$0.9D + 1.6W_{\text{tension}}$$

$$0.9D + 1.6W_{\text{compression}}$$

$$0.9D + 1.0E_{\text{tensile}}$$

$$0.9D + 1.0E_{\text{compression}}$$

Substituting numerical values will lead to:

$$1.4(850) = 1190 \text{ kN}$$

$$1.2(850) + 1.6(1100) + 0.5(250) = \boxed{2905 \text{ kN}}$$

$$1.2(850) + 1.6(250) + (1100) = 2520 \text{ kN}$$

$$1.2(850) + 1.6(250) + 0.5(-290) = 1275 \text{ kN}$$

$$1.2(850) + 1.6(250) + 0.5(350) = 1595 \text{ kN}$$

$$1.2(850) + 1.0(-290) + 1.0(1100) + 0.5(250) = 1955 \text{ kN}$$

$$1.2(850) + 1.0(350) + 1.0(1100) + 0.5(250) = 2595 \text{ kN}$$

$$1.2(850) + 1.0(1100) = 2120 \text{ kN}$$

$$0.9(850) + 1.6(-290) = 301 \text{ kN}$$

$$0.9(850) + 1.6(350) = 1325 \text{ kN}$$

$$0.9(850) + 1.0(-300) = 465 \text{ kN}$$

$$0.9(850) + 1.0(260) = 1025 \text{ kN}$$

The ultimate load on the column = 2905 kN

Example (2):

For the floor plan shown in the figure (1), if $D = 3.4 \text{ kN/m}^2$ and $L = 2.4 \text{ kN/m}^2$, find the ultimate loads on:

- Columns A4, B3, and C4
- Beams B1 – C1 and C2 – D2

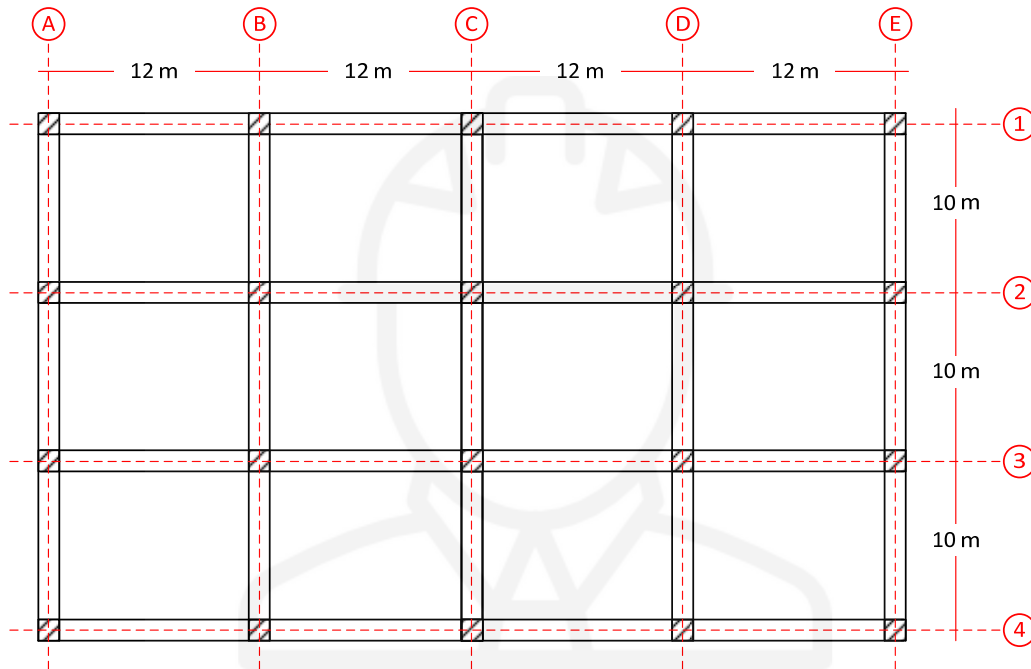


Figure 6-5: Floor plan for **Error! Reference source not found.**

Solution:**Column A4:**

$$A_T = 6 \times 5 = 30 \text{ m}^2$$

Column A4 is a corner column without cantilever slab $\rightarrow K_{LL} = 1$

$$A_I = K_{LL} \times A_T = 1 \times 30 = 30 \text{ m}^2$$

$A_I < 37.2 \text{ m}^2 \rightarrow \text{No Reduction}$

$$P_u = (1.2D + 1.6L) \times A_I = (1.2(3.4) + 1.6(2.4)) \times 30 = \boxed{237.7 \text{ kN}}$$

Column B3:

$$A_T = 10 \times 12 = 120 \text{ m}^2$$

Column B3 is an interior column $\rightarrow K_{LL} = 4$

$$A_I = K_{LL} \times A_T = 4 \times 120 = 480 \text{ m}^2$$

$A_I > 37.2 \text{ m}^2 \rightarrow$ Apply live load reduction

$$L = L_o \left(0.25 + \frac{4.57}{\sqrt{A_I}} \right) = 2.4 \left(0.25 + \frac{4.57}{\sqrt{480}} \right) = 1.1 \text{ kN/m}^2$$

$$P_u = (1.2D + 1.6L) \times A_I = (1.2(3.4) + 1.6(1.1)) \times 120 = \boxed{700.8 \text{ kN}}$$

Column C4:

$$A_T = 5 \times 12 = 60 \text{ m}^2$$

Column C4 is an exterior column without cantilever slabs $\rightarrow K_{LL} = 4$

$$A_I = K_{LL} \times A_T = 4 \times 60 = 240 \text{ m}^2$$

$A_I > 37.2 \text{ m}^2 \rightarrow$ Apply live load reduction

$$L = L_o \left(0.25 + \frac{4.57}{\sqrt{A_I}} \right) = 2.4 \left(0.25 + \frac{4.57}{\sqrt{240}} \right) = 1.31 \text{ kN/m}^2$$

$$P_u = (1.2D + 1.6L) \times A_I = (1.2(3.4) + 1.6(1.31)) \times 60 = \boxed{370.6 \text{ kN}}$$

$$L = L_o \left(0.25 + \frac{4.57}{\sqrt{A_I}} \right) = 2.4 \left(0.25 + \frac{4.57}{\sqrt{240}} \right) = 1.31 \text{ kN/m}^2$$

Beam B1 – C1:

$$A_T = \left(\frac{12+2}{2} \right) \times 5 = 35 \text{ m}^2$$

Beam B1 – C1 is an edge beam without cantilever slabs $\rightarrow K_{LL} = 2$

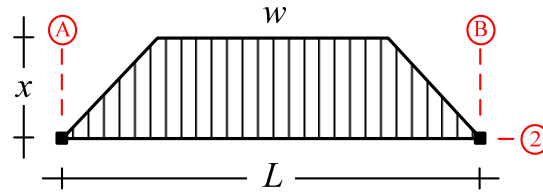
$$A_I = K_{LL} \times A_T = 2 \times 35 = 70 \text{ m}^2$$

$A_I > 37.2 \text{ m}^2 \rightarrow$ Apply live load reduction

$$L = L_o \left(0.25 + \frac{4.57}{\sqrt{A_I}} \right) = 2.4 \left(0.25 + \frac{4.57}{\sqrt{70}} \right) = 1.89 \text{ kN/m}^2$$

$$W_u = (1.2D + 1.6L) \times \perp d = (1.2(3.4) + 1.6(1.89)) \times 5 = \boxed{35.52 \text{ kN/m}}$$

Extra: Approximation of trapezoidal load as a uniformly distributed load



$$w = 35.52 \text{ kN/m}^2, x = 5 \text{ m}$$

$$\alpha = 1 - \frac{1}{3} \left(\frac{2x}{L} \right)^2 = 1 - \frac{1}{3} \left(\frac{2(5)}{12} \right)^2 = 0.769 \quad \beta = 1 - \frac{x}{L} = 1 - \frac{5}{12} = 0.583$$

Equivalent uniformly distributed load for bending moment:

$$w_{eq} = \alpha w = (0.769)(35.52) = 27.31 \text{ kN/m}$$

Equivalent uniformly distributed load for shear force:

$$w_{eq} = \beta w = (0.583)(35.52) = 20.72 \text{ kN/m}$$



Beam C2 – D2:

$$A_T = 2 \times \left(\frac{12+2}{2} \right) \times 5 = 70 \text{ m}^2$$

Beam C2 – D2 is an interior beam $\rightarrow K_{LL} = 2$

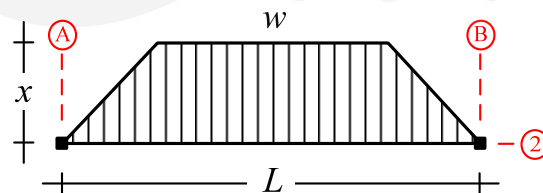
$$A_I = K_{LL} \times A_T = 2 \times 70 = 140 \text{ m}^2$$

$A_I > 37.2 \text{ m}^2 \rightarrow$ Apply live load reduction

$$L = L_o \left(0.25 + \frac{4.57}{\sqrt{A_I}} \right) = 2.4 \left(0.25 + \frac{4.57}{\sqrt{140}} \right) = 1.59 \text{ kN/m}^2$$

$$W_u = (1.2D + 1.6L) \times \perp d = (1.2(3.4) + 1.6(1.59)) \times 5 = \boxed{33.12 \text{ kN/m}}$$

Extra: Approximation of trapezoidal load as a uniformly distributed load



$$w = 33.2 \text{ kN/m}^2, x = 5 \text{ m}$$

$$\alpha = 1 - \frac{1}{3} \left(\frac{2x}{L} \right)^2 = 1 - \frac{1}{3} \left(\frac{2(5)}{12} \right)^2 = 0.769 \quad \beta = 1 - \frac{x}{L} = 1 - \frac{5}{12} = 0.583$$

Equivalent uniformly distributed load for bending moment:



$$w_{eq} = \alpha w = (0.769)(33.12) = 25.47 \text{ kN/m}$$

Equivalent uniformly distributed load for shear force:

$$w_{eq} = \beta w = (0.583)(33.12) = 19.31 \text{ kN/m}$$



Example (3):

For the 2nd floor plan shown in the figure below, assuming all slabs are 10 cm thick and:

- Concrete density (ρ) = 25 kN/m³
- Mechanical, Electrical, and Piping = 0.60 kN/m²
- Ceiling system = 0.30 kN/m²
- Roofing = 0.20 kN/m²
- Flooring = 0.35 kN/m²

1- Find the “ultimate load” on columns (A1), (B4), (C2)

2- Find the “ultimate load” on beams (A3 – A4), (C1 – C2), (B3 – C3)

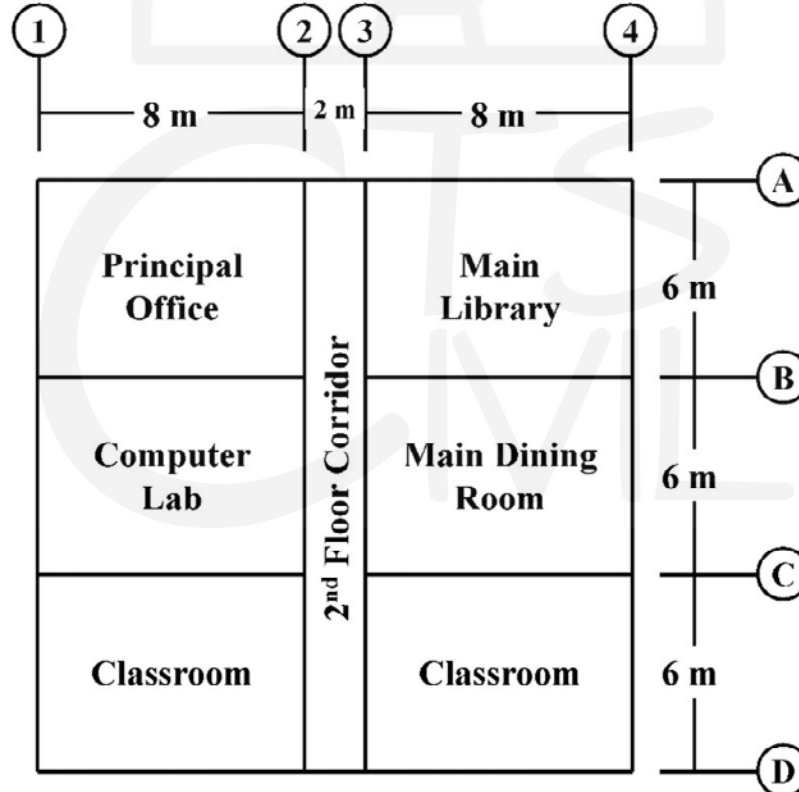


Figure 6-6: Floor plan for Error! Reference source not found.

Solution:

Dead load calculation:

$$\sum D = \left(25 \text{ kN/m}^2 \times \frac{10}{100} \text{ m} \right) + 0.60 + 0.30 + 0.35 = 3.75 \text{ kN/m}^2$$

Summary of live load values from ASCE-7:

Occupancy	Live load (kN/m ²)
Office	2.40
Computer Lab	4.79
Classroom	1.92
Corridor above first floor	3.83
Library (reading room)	2.87
Dining room	4.79

Column A1:

$$A_T = 4 \times 3 = 12 \text{ m}^2$$

Column A1 is a corner column without cantilever slab $\rightarrow K_{LL} = 1$

$$A_I = K_{LL} \times A_T = 1 \times 12 = 12 \text{ m}^2$$

 $A_I < 37.2 \text{ m}^2 \rightarrow$ No reduction

$$P_u = (1.2D + 1.6L) \times A_I = (1.2(3.75) + 1.6(2.4)) \times 12 = \boxed{100 \text{ kN}}$$

Column B4:

$$A_{T \text{ Lib}} = 3 \times 4 = 12 \text{ m}^2$$

Column B4 is an exterior column without cantilever slabs $\rightarrow K_{LL} = 4$

$$A_{I \text{ Lib}} = K_{LL} \times A_{T \text{ Lib}} = 4 \times 12 = 48 \text{ m}^2$$

 $A_I > 37.2 \text{ m}^2 \rightarrow$ Apply live load reduction

$$L = L_o \left(0.25 + \frac{4.57}{\sqrt{A_I}} \right) = 2.87 \left(0.25 + \frac{4.57}{\sqrt{48}} \right) = 2.61 \text{ kN/m}^2$$

$$P_{u \text{ Lib}} = (1.2D + 1.6L) \times A_{I \text{ Lib}} = (1.2(3.75) + 1.6(2.61)) \times 12 = \boxed{104.10 \text{ kN}}$$

Note: No reduction is allowed for dining room live load (ASCE-7)

$$A_{T \text{ Din}} = 3 \times 4 = 12 \text{ m}^2 = A_{I \text{ Din}}$$

$$P_{u \text{ Din}} = (1.2D + 1.6L) \times A_{I \text{ Din}} = (1.2(3.75) + 1.6(4.79)) \times 12 = \boxed{145.97 \text{ kN}}$$

$$\sum P_u = P_{u \text{ Lib}} + P_{u \text{ Din}} = 104.10 + 145.97 \approx \boxed{250 \text{ kN}}$$

Column C2:

$$A_{T \text{ Comp}} = A_{T \text{ Class}} = 3 \times 4 = 12 \text{ m}^2$$

$$A_{T \text{ Corr}} = 1 \times 6 = 6 \text{ m}^2$$

Column C2 is an interior column $\rightarrow K_{LL} = 4$

$$A_{I \text{ Comp}} = A_{I \text{ Class}} = 4 \times 12 = 48 \text{ m}^2$$

$$A_{I \text{ Corr}} = 4 \times 6 = 24 \text{ m}^2$$

$A_{I \text{ Comp}}, A_{I \text{ Class}} > 37.2 \text{ m}^2 \rightarrow$ Apply live load reduction

$A_{I \text{ Corr}} < 37.2 \text{ m}^2 \rightarrow$ No live load reduction

$$L_{\text{Comp}} = L_o \left(0.25 + \frac{4.57}{\sqrt{A_I}} \right) = 4.79 \left(0.25 + \frac{4.57}{\sqrt{48}} \right) = 4.36 \text{ kN/m}^2$$

$$L_{\text{Class}} = L_o \left(0.25 + \frac{4.57}{\sqrt{A_I}} \right) = 1.92 \left(0.25 + \frac{4.57}{\sqrt{48}} \right) = 1.75 \text{ kN/m}^2$$

$$L_{\text{Corr}} = L_o = 3.83 \text{ kN/m}^2$$

$$P_{u \text{ Comp}} = (1.2D + 1.6L) \times A_{I \text{ Comp}} = (1.2(3.75) + 1.6(4.36)) \times 12 = \boxed{137.7 \text{ kN}}$$

$$P_{u \text{ Class}} = (1.2D + 1.6L) \times A_{I \text{ Class}} = (1.2(3.75) + 1.6(1.75)) \times 12 = \boxed{87.6 \text{ kN}}$$

$$P_{u \text{ Corr}} = (1.2D + 1.6L) \times A_{I \text{ Corr}} = (1.2(3.75) + 1.6(3.83)) \times 6 = \boxed{63.77 \text{ kN}}$$

$$\sum P_u = P_{u \text{ Comp}} + P_{u \text{ Class}} + P_{u \text{ Corr}} = 137.7 + 87.6 + 63.77 \approx \boxed{289 \text{ kN}}$$

Beam A3 – A4:

$$A_T = \left(\frac{2+8}{2} \right) \times 3 = 15 \text{ m}^2$$

$$A_{T \text{ Corr}} = 1 \times 6 = 6 \text{ m}^2$$

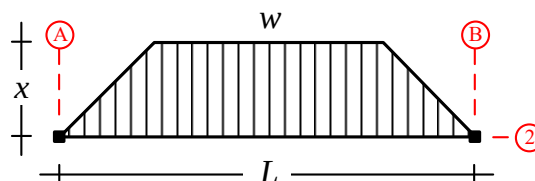
Beam A3 – A4 is an edge beam without cantilever slab $\rightarrow K_{LL} = 2$

$$A_I = 2 \times 15 = 30 \text{ m}^2$$

$A_I < 37.2 \text{ m}^2 \rightarrow$ No live load reduction

$$W_u = (1.2D + 1.6L) \times \perp d = (1.2(3.75) + 1.6(2.87)) \times 3 = \boxed{27.28 \text{ kN/m}}$$

Extra: Approximation of trapezoidal load as a uniformly distributed load



$$w = 27.28 \text{ kN/m}^2, x = 3 \text{ m}$$

$$\alpha = 1 - \frac{1}{3} \left(\frac{2x}{L} \right)^2 = 1 - \frac{1}{3} \left(\frac{2(3)}{8} \right)^2 = 0.8125 \quad \beta = 1 - \frac{x}{L} = 1 - \frac{3}{8} = 0.625$$

Equivalent uniformly distributed load for bending moment:

$$w_{eq} = \alpha w = (0.8125)(27.28) = 22.17 \text{ kN/m}$$

Equivalent uniformly distributed load for shear force:

$$w_{eq} = \beta w = (0.625)(27.28) = 17.051 \text{ kN/m}$$



Beam C1 – C2:

$$A_{T \text{ Comp}} = A_{T \text{ Class}} = \left(\frac{2+8}{2} \right) \times 3 = 15 \text{ m}^2$$

Beam C1 – C2 is an interior beam $\rightarrow K_{LL} = 2$

$$A_{I \text{ Comp}} = A_{I \text{ Class}} = 2 \times 15 = 30 \text{ m}^2$$

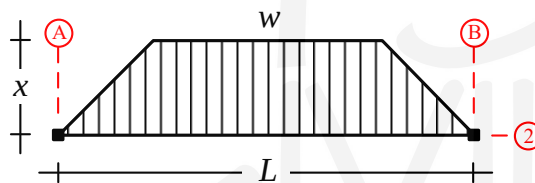
$A_{I \text{ Comp}}, A_{I \text{ Class}} < 37.2 \text{ m}^2 \rightarrow$ No live load reduction

$$W_{u \text{ Comp}} = (1.2D + 1.6L) \times \perp d = (1.2(3.75) + 1.6(4.79)) \times 3 = \boxed{36.49 \text{ kN/m}}$$

$$W_{u \text{ Class}} = (1.2D + 1.6L) \times \perp d = (1.2(3.75) + 1.6(1.92)) \times 3 = \boxed{22.72 \text{ kN/m}}$$

$$\sum W_u = W_{u \text{ Comp}} + W_{u \text{ Class}} = 36.49 + 22.72 = \boxed{59.21 \text{ kN/m}}$$

Extra: Approximation of trapezoidal load as a uniformly distributed load



$$w = 59.21 \text{ kN/m}^2, x = 3 \text{ m}$$

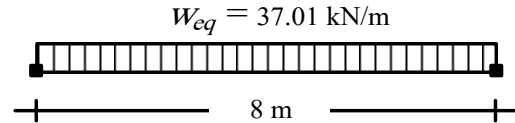
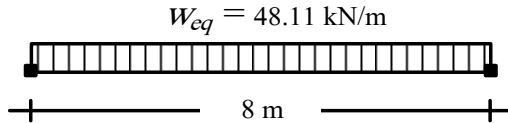
$$\alpha = 1 - \frac{1}{3} \left(\frac{2x}{L} \right)^2 = 1 - \frac{1}{3} \left(\frac{2(3)}{8} \right)^2 = 0.8125 \quad \beta = 1 - \frac{x}{L} = 1 - \frac{3}{8} = 0.625$$

Equivalent uniformly distributed load for bending moment:

$$w_{eq} = \alpha w = (0.8125)(59.21) = 48.11 \text{ kN/m}$$

Equivalent uniformly distributed load for shear force:

$$w_{eq} = \beta w = (0.625)(59.21) = 37.011 \text{ kN/m}$$



Beam B3 – C3:

$$A_{T \text{ Din}} = \frac{1}{2} \times 6 \times 3 = 9 \text{ m}^2$$

$$A_{T \text{ Corr}} = 1 \times 6 = 6 \text{ m}^2$$

Beam B3 – C3 is an interior beam $\rightarrow K_{LL} = 2$

As per ASCE-7, dining rooms $\rightarrow$ NO LIVE LOAD REDUCTION

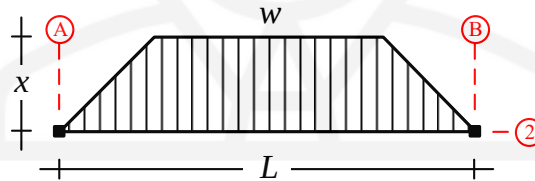
$$A_{I \text{ Corr}} = 2 \times 6 = 12 \text{ m}^2$$

$A_{I \text{ Corr}} < 37.2 \text{ m}^2 \rightarrow$ No live load reduction

$$W_{u \text{ Din}} = (1.2D + 1.6L) \times \perp d = (1.2(3.75) + 1.6(4.79)) \times 3 = \boxed{36.49 \text{ kN/m}}$$

$$W_{u \text{ Corr}} = (1.2D + 1.6L) \times \perp d = (1.2(3.75) + 1.6(3.38)) \times 1 = \boxed{10.63 \text{ kN/m}}$$

Extra: Approximation of trapezoidal load as a uniformly distributed load



$$w = 36.49 \text{ kN/m}^2 \text{ (Dining room load only), } x = 3 \text{ m}$$

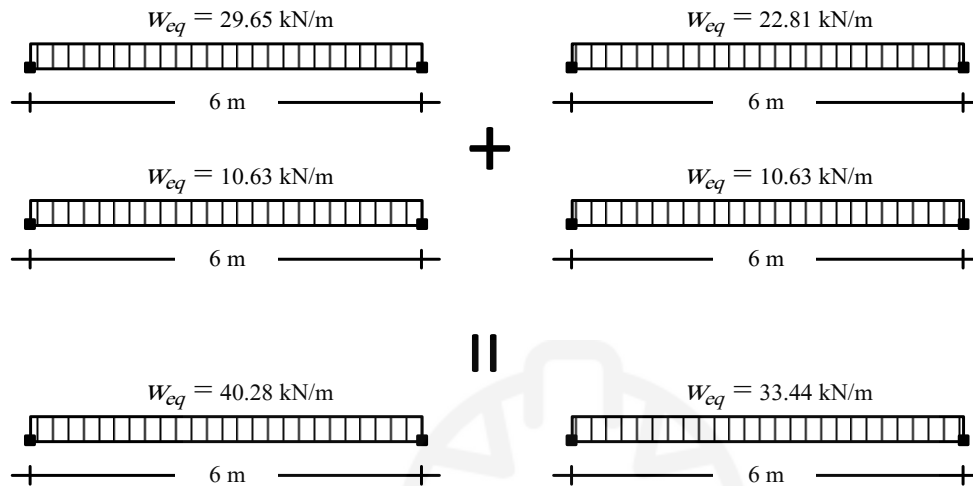
$$\alpha = 1 - \frac{1}{3} \left(\frac{2x}{L} \right)^2 = 1 - \frac{1}{3} \left(\frac{2(3)}{8} \right)^2 = 0.8125 \quad \beta = 1 - \frac{x}{L} = 1 - \frac{3}{8} = 0.625$$

Equivalent uniformly distributed load for bending moment:

$$w_{eq} = \alpha w = (0.8125)(36.49) = 29.65 \text{ kN/m}$$

Equivalent uniformly distributed load for shear force:

$$w_{eq} = \beta w = (0.625)(36.49) = 22.81 \text{ kN/m}$$



6.4 Problems:

Question:

For the floor plan shown in Figure (1), assuming all slabs are 10 cm thick and:

- Concrete density (ρ_c) = 25 kN/m³
- Mechanical, Electrical, and Piping = 0.5 kN/m²
- Ceiling system = 0.25 kN/m²
- Roofing = 0.28 kN/m²
- Flooring = 0.40 kN/m²

1. Find the “ultimate load” on columns (A1), (D3), and (E5).
2. Find the “ultimate load” on beams (A1–A2), (E5–E6), (C4–E4) and (A6–C6).

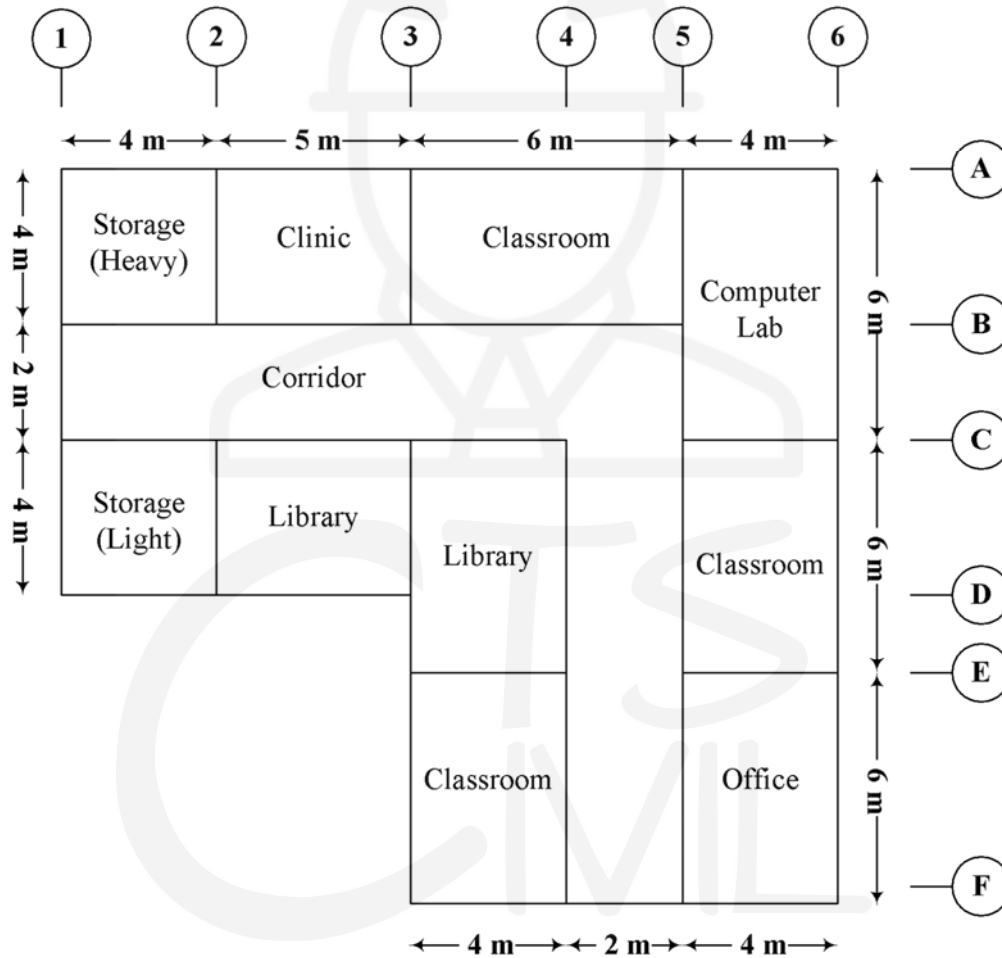


Figure 1: Partial school floor plan (2nd floor)